An easy way to Physics: 05 Mechanical Energy

Potential Energy

We take a mass m and put it on a cardboard box. We lift the mass to the height h.
We let the mass fall down, it becomes faster and faster. Finally it hits the box and destroys it.
Why did the mass become so naughty?
We put something into the mass by lifting it. We put Energy E into the mass.
In order to lift this mass, we needed a force $F_{lift} = m \cdot g$.
The potential energy $E_{pot} = F_{lift} \cdot h = m \cdot g \cdot h$ is the product of force times distance covered in the direction of the force.

The unit of energy is 1 J [Joule] = 1 kg · m² / s².
I have to invest 1 J if I lift a mass of about 0,1 kg by 1 m.

Energy in a Spring

When you compress or extend a spring by the distance x, you put potential energy into it. The force we need, however, is not constant, but increases with x: $F_{spring} = k \cdot x$.
In order to calculate the potential energy at the compression x, we need the average force between 0 and x: $\overline{F}_{spring} = ½ \cdot k \cdot x$.
The potential energy of a spring is then $E_{pot\ spring} = \overline{F}_{spring} \cdot x = . ½ \cdot k \cdot x \cdot x .= ½ \cdot k \cdot x^2$

Work in Physics

The increase of energy achieved by pushing something a distance s with a force F is called work in physics: $W = F \cdot s$

Kinetic Energy

When we lift up the mass, we put potential energy into it. When the mass falls down, it loses its potential energy and the mass gets a velocity v. We know already: $h = ½ \cdot g \cdot t^2$ and $v = g \cdot t$.
$E_{pot} = F_{lift} \cdot h = m \cdot g \cdot h = m \cdot g \cdot (½ \cdot g \cdot t^2) = ½ \cdot m \cdot g \cdot g \cdot t^2 = ½ \cdot m \cdot v^2$
All of the potential energy is converted to the energy of movement or
kinetic energy $E_{kin} = ½ \cdot m \cdot v^2$

A car with mass = 800 kg and a velocity of 10 m/s (36 km / h) has a kinetic energy of
$E_{kin} = ½ \cdot m \cdot v^2 = ½ \cdot 80\ kg \cdot (10\ m/s)^2 = 4\ 000\ J$.
If the car goes with 20 m/s, its kinetic energy is $E_{kin} = ½ \cdot 80\ kg \cdot (20\ m/s)^2 = 16\ 000\ J$.

If a meteorite with m = 1000 kg hits the earth with v = 10 000 m/s, its energy is 100,000,000,000 J
If a star collapses in a Supernova explosion, it releases an energy of 10^{46} J. (a 1 with 46 zeroes.)

When the mass hits the cardboard box, the kinetic energy of the box is transformed into the energy needed to destroy the box.

Conservation of Energy

In our example of the falling mass there are different forms of energy. There are transitions from one kind of energy to the other. The sum of the energies, however, remains constant, if no additional energy comes from the outside and if no energy is given away to the outside.

Helpful Applications of the Conservation of Energy

1) The Inclined Plane

To lift a mass m to the height h, I can lift up the mass vertically or over an inclined plane. In both cases I have to invest the energy $E = m \cdot g \cdot h$. In the vertical movement I have the force $F = m \cdot g$. On the inclined plane I have to push a much longer way s. $F_{plane} \cdot s = m \cdot g \cdot h$. => $F_{plane} = m \cdot g \cdot h/s$. That means, that I need much less force than before. That is also the way the thread of a screw works: By the thread I can increase the vertical force.

2) The Lever

Look at a horizontal lever. Its left end rests on a hard surface, and your hand holds it at the right end with a distance r_1 from the left end. A mass m hangs on the lever at a distance r_2 form the left end. When you lift the right end of the lever by a small distance s_1, then the mass moves upward by the distance $s_2 = s_1 \cdot r_2 / r_1$. If you lift up the right end with a force F_1, the mass experiences a force F_2. As $F_1 \cdot s_1 = F_2 \cdot s_2$, $F_2 = F_1 \cdot s_1 / s_2 = F_1 \cdot r_1 / r_2$. The quantity $T = F_1 \cdot r_1 = F_2 \cdot r_2$ is called the torque. Thus by choosing the distances r you can increase the force by a lever.

3) The Pulley

If we bind a mass m to a string, we have to pull the string with the same force as the mass directly. If we let the string run over a reel fixed on the ceiling, we change only the direction of the force but not its size.

If we, however, fix a reel on the mass and the one end of the string on the ceiling, and if we want to lift the mass by a distance h, we have to lift the free end of the string two times the height h, because we have to shorten two pieces of string by h. In this time we lift by half of the force only.

In a pulley we have to shorten 4, 6 or 8 pieces of string by h. That means I have to take much more rope, the force I must employ, however is only 1/4, 1/6 or 1/8.