Measurements

Physics wants to understand reality and its structures. It starts with observations and ideas. Then it makes measurements to check and improve the ideas it has. In an experiment measurements are set up in a certain way in order to see certain effects very clearly and to minimize other influences. The results of the measurement lead to a better description of nature. The results of thoughts and measurements lead to a thought structure, called a theory. A theory allows us to predict things, that are not measured up to now. New experiments allow then to check the theory and to modify it if needed, or even to abandon it.

Measurement errors

When we measure a certain quantity, for example a length, we find out, that repeated measurements often don't give the exact same result, and that our measuring device allows for a limited accuracy only. With a ruler, we can achieve an accuracy not better than 1 mm. This limited accuracy is caused by many unknown influences. If we measure a length of 247 mm, than this measurement means, that the true value of this length is greater or equal 247-1 mm = 246 mm and less or equal 247+1 mm = 248 mm. Therefore it makes no sense to show the result with additional digits, for example 247, 33785 $\pm$ 1 mm. The correct result is: 247 $\pm$ 1 mm. For each measured value, we need the numerical value, the estimate of the error of the measurement and the unit. If l is the symbol for length, then the error of $\pm$ 1 mm is called the absolute error and has the symbol Δl. The relative error $\delta l = \Delta l / l$ gives the error as a percentage of the result.

If a number of different quantities are at work, what is the error of my result?

If all the unknown influences, that disturb my measurements, can be both positive or negative, and if we know x, Δx, y and Δy, then the following rules apply:

$$\Delta(x + y) = \Delta(x - y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\delta(x \cdot y) = \delta(x / y) = \delta(x) \cdot \delta(y)$$

Accuracy and Precision

$$\Delta(x + x + x) = \sqrt{(\Delta x)^2 + (\Delta x)^2 + (\Delta x)^2} = \sqrt{3(\Delta x)^2} = \sqrt{3}\,\Delta x$$

$$\delta(x + x + x) = \Delta(x + x + x) / (x + x + x) = \sqrt{3}\,\Delta x / (3\ x) = \Delta x / (\sqrt{3}x) = \delta(x) / \sqrt{3}$$

If measurements are repeated, the relative errors become smaller. The precision of a value indicates, how close the results of repeated measurements are to each other.

If, however, the ruler used for the measurement is 2 mm too short, the measurement will always be too short, no matter, how often I repeat the measurement. I get a bias. The accuracy informs about the fact, how far the measured value is away from the real value. High accuracy means, that the results of the measurement are very close to the real values. The accuracy can be increased, when well known lengths are measured and the measurements are compared to the real values. From this comparison I can derive, how I have to correct my measured values.

Stating Results

Well documented results of a measurement should contain the following items:

1) The raw data, that are the measured values,
2) the corrections and why they were used,
3) the corrected values, and
4) an estimate of the accuracy of the corrected values